175 §2- CALCULUS II - Self Test - II

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Name ____

You do not have to turn this in, the goal is to help you identify areas that may require some attention. If there are any problems you are curious about, we can talk about them in office hours. Do not spend longer than 5 minutes per question (except perhaps for the last one), and aim at spending no longer than the number of minutes indicated in parentheses at the beginning of each question. Only work on the last problem if you have finished the others. **Do not use calculators** and if possible, try to remember the required formulas rather than consulting them in the book or notes.

1. (≤ 1) Find the derivative of $-\dfrac{1}{3}\ln(x^3+8)$.

   [Do **not** forget the inner derivative.]

2. (≤ 1) Find the derivative of $-\dfrac{2}{3}\ln(x^2-2x+4)$.

3. (≤ 2) Find the derivative of $-\dfrac{4}{3}\sqrt{3}\tan^{-1}\left(\dfrac{x-1}{\sqrt{3}}\right)$.

   [Your answer should be $-\dfrac{4}{x^2-2x+4}$.

   `Hint:` If you don't remember how to take the derivative of $\tan^{-1} x = y$, use inverse functions to get $x = \tan y$, and use implicit differentiation and some trig. identities. This should still take less than 5 minutes.]

4. (≤ 1) Factor $x^3 + 8$.

5. (≤ 4) Verify whether the following is correct:

$$\int \frac{x^4 - x^2}{x^3 + 8}\, dx = -\frac{1}{3}\ln(x^3+8)+\frac{1}{2}x^2-\frac{2}{3}\ln(x^2-2x+4)+\frac{4}{3}\ln(x+2)-\frac{4}{3}\sqrt{3}\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right)+C.$$

6. (≤ 2) Use the substitution $u = x^2 - 9$ to solve

$$\int \frac{x\, dx}{\sqrt{x^2 - 9}}.$$

7. (≤ 3) Use a trig. substitution to solve

$$\int \frac{x\, dx}{\sqrt{x^2 - 9}}.$$

[Of course, your answer should be the same as above.]

8. (≤ 2) Use the substitution $u = x^2 - 9$ to solve

$$\int \frac{x\,dx}{x^2 - 9}.$$

9. (≤ 4) Use a trig. substitution to solve

$$\int \frac{x\,dx}{x^2 - 9}.$$

[`Hint:` Recall that $\sec^2\theta = \tan^2\theta + 1$ and that $1/\tan\theta = \cot\theta$.]

10. (≤ 3) Use partial fractions to solve

$$\int \frac{x\,dx}{x^2 - 9}.$$

11. (≤ 10) Weierstrass substitution $x = \tan(\theta/2)$ is useful when integrating certain trigonometric expressions for which the techniques we saw may not suffice.

As an example, use it to solve

$$\int \frac{d\theta}{1 + \sin\theta}.$$

In order to do this:

- Show that $d\theta = \dfrac{2\,dx}{1 + x^2}$.
- Show that $\sin\theta = \dfrac{2x}{1 + x^2}$. For this, first verify the following chain of equalities:

$$\sin\theta = 2\sin(\theta/2)\cos(\theta/2) = 2\tan(\theta/2)\cos^2(\theta/2) = \frac{2\tan(\theta/2)}{\sec^2(\theta/2)}.$$

[This integral can also be done with the techniques we have studied. Try to see how you would evaluate it.]