175 §2- CALCULUS II - Self Test

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Name _____

These questions are divided into four groups. Ideally, you would answer YES to all questions in group A, to most questions in group B, and to about half of the questions in group C. Imagine that no calculators are allowed, unless otherwise specified. (Any question you answer NO to is a topic you may want to try to improve.)

Group A (Answer Yes/No/Depends next to each question, as applicable. If you answer Depends, give a quick explanation.)

1. You can solve systems of 2 linear equations in 2 unknowns very quickly and without difficulties.

2. You can solve systems of 3 linear equations in 3 unknowns in a minute or two.

3. You can solve systems of 4 linear equations in 4 unknowns in less than 5 minutes.

4. From the graph of a function in a given interval you can quickly recognize whether the inverse function exists.

5. From the graph of a function whose inverse exists you can quickly sketch the graph of the inverse function.

6. You can compose functions quickly without difficulties (e.g, if $f(x) = \tan(x-3) + \sqrt{x}$ and $g(x) = \ln(3x-5)$ then you have no problem writing formulas for $f(g(x)), g(f(x)), f(f(x))$, etc.)

7. You can manipulate logarithms without difficulties.

8. You know by heart basic trigonometric identities.

9. You can easily add two or more quotients of polynomials.

10. You can quickly work with inequalities without major difficulties.

11. You can easily factor polynomials with integer roots.

12. You can quickly complete squares.

13. You know by heart the quadratic formula.

14. You can find the distance between two points without difficulties.

15. You can find the distance between a point and a line. (If yes, estimate how long you expect this would take you.)

Group B (Answer Yes/No/Depends next to each question, as applicable. If you answer Depends, give a quick explanation.)

1. You can easily compute limits of polynomial expressions.

2. You can easily compute limits of quotients of polynomials.

3. You can quickly compute limits of trigonometric functions.

4. You can quickly compute derivatives and integrals of polynomials.

5. You can quickly compute derivatives and integrals of powers.

6. You can differentiate trigonometric expressions without difficulties.

7. You know by heart the product and quotient rules.

8. You know by heart how to take derivatives of composite functions.

Group C (Answer Yes/No/Depends next to each question, as applicable. If you answer Depends, give a quick explanation.)

1. You know how to set up integrals to compute volumes of solids of revolution without difficulties (independently of whether you can compute the corresponding integrals).

2. You understand how to obtain the partial fractions decomposition of a quotient of polynomials (independently of whether you can do the corresponding algebra or not).

3. You understand when to apply trigonometric substitutions to solve integrals, and have no problem knowing which kind of trigonometric substitution is appropriate for a given expression.

4. You can apply the method of integration by parts without problems.

5. You understand why integration by parts works (independently of whether you know how to apply the method or not).

6. At least in principle, you can apply the method of integration by parts to obtain reduction formulas (such as the formulas we obtained in lecture for $\int x^n e^x \, dx$ or $\int \sec^n x \, dx$).

Group D (Do not actually compute the integrals below.)

1. Explain how you would verify whether the following is correct.

$$\int \frac{x^4 - x^2}{x^3 + 8} \, dx = -\frac{1}{3}\ln(x^3+8) + \frac{1}{2}x^2 - \frac{2}{3}\ln(x^2-2x+4) - \frac{4}{3}\sqrt{3}\tan^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + C.$$

2. What trig. substitution (e.g., of the form $x = a\tan\theta$ or $x = a\sec\theta$, or $x = a\cos\theta$ or something else) would you use to solve

$$\int \frac{dx}{x\sqrt{x^2-9}}?$$

3. How would you solve the integral below? (E.g., "Parts and then a trig. substitution" or "a trig. substitution" or "partial fractions" or something else.)

$$\int \frac{dx}{x^2 - 1}.$$

4. Same for

$$\int \frac{dx}{x^4 - x^2}.$$

5. If you were asked to compute numerically the value of $\sqrt{2}$, what kind of strategy would you use?

6. Same for e.

7. Are there any topics covered in Calculus I you wish you understood better?

8. Of the methods of integration we covered, which one(s) do you understand best? Which one(s) do you understand least? If algebraic manipulations are not a problem, does your answer change in any way?