187 §1- DISCRETE MATHEMATICS - Midterm 1

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Name _____

Answer on your blue book. Indicate clearly in each page the problem you are working on. Make sure to show all your work. Do not skip any non-obvious steps.

1. In all cases, please explain how you found your answer. Find the number of lists of length 2 whose elements are chosen from $\{0, 1, 2, 3, 4\}$ and:

   (a) Repetitions are allowed.

   (b) Repetitions are required.

   (c) Repetitions are not allowed.

   [**If you have finished the whole exam**, and only in that case, and if you want an extra credit problem, count the number of lists of length 3 whose elements are chosen from $\{0, 1, 2, 3, 4\}$ and repetitions are required.]

2. Explain in what sense the statement $(\varphi \Leftrightarrow \psi)$ is equivalent to the statement $((\neg\varphi) \Leftrightarrow (\neg\psi))$, and prove that this is the case.

3. Suppose A, B, C are sets, and x is an element such that $x \in (A \triangle B)$ and $x \in C$. Show that $x \in \big((A \cap C) \triangle (B \cap C)\big)$.

4. Prove in detail: If $a|b$ and x is divisible by b, then a is a factor of x.

5. How many positive divisors does the number $3 \times 4 \times 5 \times 6 \times 7$ have? Explain carefully how you are doing the counting.